

COLLECTIVE BEHAVIOUR DETECTION OF STOCK MARKETS: EVIDENCE FROM 2007-2008 CRISIS

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ABSTRACT

The inference of interconnectivity structure between different stochastic processes is of particular interest, in many fields such as the study of stock markets. A typical assumption in the estimation process of causal inference is stationarity, which violates the applicability of existing statistical analysis techniques. For this purpose, the most recent numerical algorithm called Auto-Regressive Estimation (ARE) algorithm has been successfully applied to EEG Brain data for detecting diseases. This manuscript introduces its application, for the first time, to stock market data. The applications, here, can contribute to a substantially deeper understanding of stock market dynamics and interdependencies before, during and after the crisis. This provides investors and market players with valuable information, which enables them to efficiently selecting less risky portfolios. From this, I argue that the utilised algorithm provides novel information regarding the detection of early warnings, which previous approaches could not assess. Furthermore, the distinction between markets that will fall into a crisis and the ones that will not, was evidenced.

KEYWORDS: Expectation-Maximisation algorithm, Financial markets, Interconnectivity structure, Kalman filter, State-Space Modelling

1. INTRODUCTION

Several statistical analysis techniques have been developed to detect interrelationships in multivariate systems. Examples of such techniques are based on mutual information (Frenzel, 2007; Paluš and Stefanovska, 2003; Pompe et al., 1998), autoregressive processes (Dahlhaus and Eichler, 2003; Eichler, 2001), coherence (Halliday and Rosenberg, 2000; Dahlhaus, 2000; Nolte et al., 2008) and recurrence in state space (Chicharro and Andrzejak, 2009; Romano et al., 2007).

A typical assumption to be made when estimating the causal structure of measured data, is stationarity. However, the underlying stock market behaviour is, in fact, governed by time-dependent dynamics and non-stationarity is present. This violates the assumptions underlying the standard techniques, that are usually based on the concept of Granger causality, that was originated from Econometrics (Granger, 1969; Granger, 1988; Granger, 1963; Granger, 1980). The most well known frequency and time domain techniques based on this concept are, renormalized partial directed coherence (rPDC) (Schelter, Timmer and Eichler, 2009) and directed partial correlation (DPC) (Eichler, 2005), respectively.

There was a widely used approach, that allows for making the underlying stock markets dynamics being almost stationary, which was based on taking the logarithm of stock market returns, instead of working with prices, i.e. raw data (Bessler and Yang, 2003; Glezakos, Merika and Kaligosfiris, 2007; Égert and Kočenda, 2007; Martens and Poon,

2001; Sandoval, 2014; Sandoval and Franca, 2012). In addition, the moving window approach was a way to allow time dependent parameters, to be estimated (Elsegai, 2015). However, the main problem associated with this approach is the appropriate choice of the time window length, as well as, whether there is an overlap or a non-overlap between these windows (Sandoval, 2014; Elsegai, 2015). In general, if the window size was not chosen properly, then this might lead to an under or an overestimation of the inferred interactions (Elsegai, 2015). In this case, therefore, other approaches should be used. A recent effective numerical algorithm, that was developed for overcoming these two problems was suggested in, Schelter et al., 2014. The authors presented a mathematical framework, to uncover the time-dependent interaction structure from measured data of arbitrary non-stationary, stochastic models. The aim of this approach, was to estimate the model parameters, without using windowing approach, avoiding potential problems associated with it. This provides a deeper understanding of the dynamics and the interactions inherited in such systems (Schelter et al., 2014). The main idea behind this algorithm was based on state-space modelling (SSM) (Stoffer and Shumway 2000), using Expectation-Maximization (EM) algorithm (Dempster, Laird and Rubin, 1977), together with Kalman Filter (KF) (Kalman, 1960). This algorithm has been successfully applied to Electroencephalography (EEG) data (Sommerlade et al., 2015) and Magnetoencephalography (MEG) data (Gao et al., 2015).

In this manuscript, the algorithm was applied to international stock market time series data. That is, for the purpose of the reconstruction of time-dependent stock market interaction parameter spaces. This shows how the topology of the resultant interaction parameter spaces, changes from a non-crisis state into a state in which the crisis occurred. This gives a clear warning sign that confirms the tendency towards a potential crisis. That, in turn, makes the investors manage their losses, before their occurrence. In addition, this analysis enables investors and market players, to differentiate between companies and markets, which will fall in a catastrophic loss from the ones which will not. As a consequence, they can take successful decisions towards selecting less risky portfolios, which guarantees less loss occurrence.

This manuscript was structured as follows. The methods that were applied in this work, which consist of State Space Model and Expectation-maximization algorithm, together with Kalman Filter, are presented in Section 2. In Section 3, the applications of the methods to international stock markets are discussed.

2. METHODOLOGY

This section presents the methods used in this manuscript, that are applied to stock market time series data in section 4. The work is based on a method, that is called ARE which stands for Auto-Regressive Estimation. This algorithm, is a combination of the State Space Model (SSM), which is presented in the first subsection, while, Expectation-maximization (EM) algorithm together with Kalman Filter (KL), is introduced in the second subsection. In addition, for model order selection for SSM, an AICi is briefly discussed in the third subsection.

2.1 State Space Model (SSM)

The State Space Model (SSM), is a method for modelling both observed and hidden processes in a given system. Such a model contains two equations. The first equation models the dynamics or the state of the process and the Gaussian distributed, driving noise and it is called the state equation. The second equation models the observations with Gaussian distributed observational noise and it is called the output equation (Mader et al., 2014).

An SSM can be written as (2010; Mader et al., 2014):

$$\begin{aligned} X(t) &= A X(t-1) + \varepsilon(t) \\ Y(t) &= C X(t) + \eta(t). \end{aligned} \quad (1)$$

The first equation describes the dynamics of the process $X(t)$ which is modelled by a vector autoregressive process (VAR [p]) and $\varepsilon(t)$ is a Gaussian driving noise with zero mean and covariance matrix Q . In addition A , which is the transition matrix, and Q are constant matrices. The observation of the process is described by the second equation $Y(t)$, where C is the observation matrix and $\eta(t)$ is the observational noise which is assumed to be Gaussian distributed with zero mean and covariance matrix R .

2.2 Expectation-maximization (EM) Algorithm and Kalman Filter (KL)

For the estimation of the state-space model parameters, Expectation-maximization (EM) algorithm and Kalman filter are used (Schelter et al., 2014). The Expectation-maximization (EM) algorithm is based on an iterative scheme, which consists of two steps. In the expectation-step, conditional expected values of the hidden states $X(t)$ and its covariance $P(t)$ are obtained, using the Kalman filter based on the equations explained above. In the maximization step, based on these values, the expected value of the likelihood is maximized with respect to the parameters which results in a new set of parameters, which is used in the next iteration of the EM algorithm (Mader et al., 2014). In the first iteration of the EM algorithm, the parameter $P(1)$, need to be initialized. Therefore, for instance, the least squares parameter estimates can then be used.

In other words, the expectation maximization (EM) algorithm, provides an iterative maximum likelihood estimator, for the parameters in the State Space Model (SSM) (Dempster, Laird and Rubin, 1977). This EM algorithm for SSM, is based on the so-called Kalman filter (Kalman, 1960). This filter is utilized to get estimates of the hidden states. The state estimates are then used to improve the estimates of the process parameters (Schelter et al., 2014; Mader et al., 2014).

To introduce Kalman filter, a measurement time series containing n observations is assumed with time and is used for the reference of these observations (Schelter et al., 2014; Mader et al., 2014). For conditional expectations (Schelter et al., 2014)

$$X^s(t) = E[X(t) | Y(1), \dots, Y(s)] \quad (2)$$

$$\begin{aligned} P^s(t_1, t_2) &= E[(X(t_1) - X^s(t_1)) - (X(t_2) - X^s(t_2))^T] \\ &= E[(X(t_1) - X^s(t_1)) - (X(t_2) - X^s(t_2))^T | Y(1), \dots, Y(s)] \end{aligned} \quad (3)$$

The subscript denotes the estimation time point, while the superscript is up to which measurement is conditioned on. The equality in Equation (2) holds, if the underlying process is Gaussian, which is assumed here. The Kalman filter is described in terms of a set of equations, that is based on an effective recursive computational way, to estimate the state of the SSM process, which minimizes the mean of the squared error.

The Kalman filter equations are (Mader et al., 2014):

$$X^{t-1}(t) = A X^{t-1}(t-1) \quad (4)$$

$$X^t(t) = X^{t-1}(t) + K(t)(Y(t) - C X^{t-1}(t)) \quad (5)$$

$$P^{t-1}(t) = AP^{t-1}(t-1)A^T + Q \quad (6)$$

$$P^t(t) = P^{t-1}(t) - K(t)CP^{t-1}(t) \quad (7)$$

$$K(t) = P^{t-1}(t)C^T(CP^{t-1}(t)C^T + R)^{-1} \quad (8)$$

With initial values and $P^0(0) = E[X(t).X(t)^T] = \Sigma$. The idea of how Kalman filter works are based on a recursive cycle of a timely update and a measurement update step (Bishop and Welch, 2001). The time update, in Equations (4) and (6), predicts the state from time to t , which results in the prior estimate $X^{t-1}(t)$ and its covariance $P^{t-1}(t)$ (Mader et al., 2014). The measurement update step consists of Equations (5) and (7) and it corrects the prior estimates, by taking into account the current prediction $X^{t-1}(t)$, the measurement $Y(t)$ and the Kalman filter gain, in Equation (8), and this leads to the posterior estimates (Mader et al., 2014). The Kalman filter equations apply a recursive scheme, as only the observations and estimates from the past and the present. Applying the steps of EM algorithm, together with Kalman filter iteratively, ensures convergence to the best estimator of the underlying dynamics and the parameters of the process (Schelter et al., 2014).

2.3 Akaika Information Criterion (AIC) for Model Order Selection

In time series applications, an appropriate model order must be chosen to characterise the collected data. The most standard criterion of scientific theory for this determination is the so-called Akaika Information Criterion (AIC) that was introduced by (Akaike, 1974). This criterion provides an insight about how a fitted model is close to the underlying generating model. However, this approach might suit some models but not all. For State Space Models, therefore, an extended AIC was developed to match the requirements of such models and give more accurate model order selection. This extended AIC is called AIC_i, where “i” refers to “improved” (Bengtsson and Cavanaugh, 2006), which is utilised here in this manuscript.

3. APPLICATIONS TO STOCK MARKET TIME SERIES DATA

This section is divided into two subsections. In the first subsection, the method is applied to 15 international stock markets. In the second subsection, the method is applied to US S&P500 Index.

3.1 Application to 15 International Stock Markets

3.1.1 Data Collection

The data used here consists of 15 international daily stock market indices at closing time. More specifically, the study includes 4 American markets, 5 European markets and 6 Asia-Pacific markets. The indices, with respective markets are displayed in Table 1. The data are collected from the Yahoo Finance database at <http://finance.yahoo.com>. The analysed sample period is from 2006 to 2010 and is divided into three periods: pre-crisis (2006-2007), crisis (2008-2009) and post-crisis (2010). The analysis covers the whole period with a total of 1417 observations for each data set.

Table 1: International Stock Markets with Respective Indices

World Markets	Country	Index
American	US	S&P500
	Brazil	Bovespa
	Canada	S&P/TSX 60
	Mexico	Didier Domi
European	UK	FTSE 100
	Europe	Euronext 100
	France	CAC 40
	Germany	DAX
	Switzerland	SMI
Asia-Pacific	Australia	S&P/ASX 200
	China	Shanghai-SE-Composite
	Hong Kong	Hang Seng
	India	S&P BSE Sensex
	Japan	Nikkei 225
	Taiwan	TSEC 50

3.1.2 Data Pre-Processing

When analyzing stock market prices, that are collected from markets in different geographic regions, one should take the differences in weekends and public holidays, as well as differences in time zones between countries into consideration. For the differences in weekends and public holidays, I match the indices at the same calendar dates. Meaning that, when one of the two markets is not traded on a specific day, the price of that day is removed from the analysis. This results in a total of 1265 observations, out of 1417 observations for each data set. For the differences in time zones, the ARE algorithm used here overcomes this problem. This is because that, the objective of the described methods is to observe the pattern and the tendency of the movements of the markets, to differentiate the different crisis states. In other words, the aim is not to draw interaction directed arrows, between every market and others as discussed in Elsegai, 2015, but to observe the general pattern of the flow of the markets and how the markets move from one state to another, over the years (2006-2010). This tool, i.e., ARE, is ideal for knowledge discovery in datasets, because it determines grouping structure in time series data (Schelter et al., 2014; Sommerlade et al., 2012; Mader et al., 2014; Gao et al., 2015).

Table 2: AIC_i Order Selection Results. It shows that Optimal Order for the SSM is 3

World Markets	AIC _i
1	58
2	55
3	689
4	40
5	25
6	0
7	0
8	0
9	0
10	0

3.1.3 Model Order Selection

In order to draw accurate conclusions, the model order must be chosen correctly. For State Space Model order selection, the “improved” Akaike Information Criteria (AIC_i), that was proposed in (Bengtsson and Cavanaugh 2006) is

utilised here. The results in Table 2 show that, the optimal chosen order is 3 for the estimation process of the State Space Model.

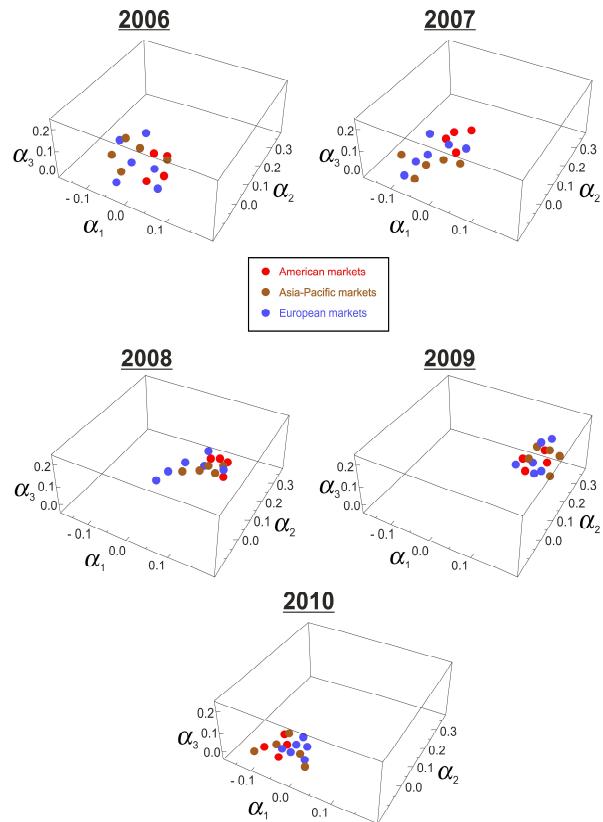


Figure 1: Time-Dependent Parameter Spaces Represent Stock Markets Behaviour during Three Periods; Pre-Crisis (2006-2007), Crisis (2008-2009) and Post-Crisis (2010). In other Meaning, the Parameter Values that Describe the Systems of the Stock Markets Change Over the Period 2006-2010. The Markets are shown as Cooled Points as Follows: Red Points (American Markets), Brown Points (Asia-Pacific Markets) and Blue Points (European markets).

3.1.4 RESULTS

Here, the numerical algorithm ARE is applied to the 15 international stock market time series data. The parameter spaces, estimated using the numerical algorithm are shown in Figure 1. The points are represented in three different three colours; red points present 4 American markets, brown points show 5 Asia-Pacific markets and blue points refer to 6 European markets. In addition, the points describe the state of the respective systems, i.e., the stock markets. The coordinates of each point are leading three estimated autoregressive parameters, for every point of time, which are α_1 , α_2 and α_3 . The 3-dimentional parameter spaces are shown as snapshots, i.e., time frames, representing the motion and the behaviour of the markets over each year separately. The estimation process results in a sequence of different sets of parameter values, that describe the state of each point in the parameter space.

The results obtained are fully consistent with the history of the 2007-2008 crisis. Therefore, the results in Figure 1 can be interpreted as follows. In year 2006, it can be observed that, the markets were away from each other and moving in a distributive way. However, there was a potential direction of a collective motion of all the markets, in the year 2007.

By taking a closer look into the parameter space, that represents the markets' motion in the year 2007, it can be shown that, the American markets, shown in red, took the lead of the observed motion and followed by some of the Asia-Pacific and European markets, shown in Brown and blue, respectively. This collective motion is evidenced as it is shown in the parameter space, representing the year 2008. From this, a pattern can be recognized as the markets move collectively towards another different state over time, in the parameter space that is represented as a snapshot, taken in the year 2009. Note that in the year 2010, the markets moved back to the initial state, where there was no crisis. This analysis shows the importance of determining, which markets will fall into the crisis and which will not, in order to give a warning sign to the investors, to protect their investments away from a potential catastrophic loss.

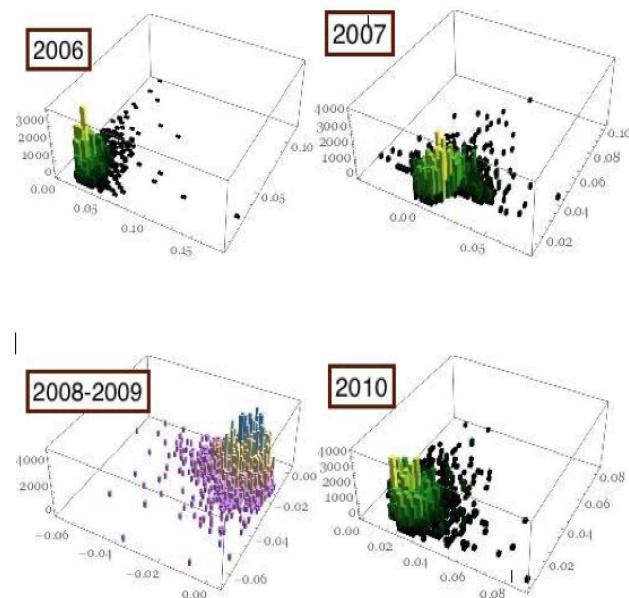


Figure 2: The Resulting Estimated Parameter Spaces Representing the Collective Motion of the Most Representative 500 US Companies in American Markets. The Results are Shown as a 3-Dimensional-Histogram Showing that there are, Approximately, Around 5% of the Markets are Not Interconnected with Others that are Falling Into the Crisis

3.2 Application for US S&P500 Index

In order to confirm the idea of the results obtained in Section 3.1, in this subsection, ARE algorithm is applied to the S&P500 Index, representing the most representative 500 US companies that belong to American stock markets. This analysis will give a deep insight about the idea of the collective motion of a large number of companies. More precisely, the aim of this analysis is to provide more evidence about the efficiency of the ARE algorithm, in pattern recognition in stock market time series datasets.

The resulting estimated parameter spaces are shown in Figure 2. According to the observation in Section 3.1, it can be asserted that in the year 2006, the companies were in the state, where there was no potential crisis that existed. However, it is obvious that there were some companies, which do not belong to the cluster of the other companies. In the year 2007, it can be seen that not the whole cluster moves collectively, but there were some companies that move individually, in different directions. This result shows that, approximately, around 95% of the companies were

interconnected and not all were influencing each other. Interestingly, in year 2008-2009, where the crisis reached its peak, it can be observed that all companies moved to another different state except some of them. Note that, the points of the 3-dimensional histogram is in yellow, green and black in the state in which the markets are not fully falling into a crisis, that is shown in the parameter spaces of the years 2006-2007, as well as, in the year 2010. However, in the parameter space that represents years 2008-2009, it can be observed that the color is totally changed, and this is the state where the crisis occurred and reached its peak.

These results assert that, differentiation of companies based on its ability to withstand crisis can be made. This is very important for investors to know and keep in their records, to take their cautions before a certain crisis occurs. Finally, all companies went back to the initial state, when the situation in the financial market settled down, to see the parameter space represents the market behaviour in year 2010.

4. CONCLUSIONS

Uncovering relationships are of particular interest, in the analysis of multivariate systems. Studying stock markets interconnections are valuable for market players, as well as, for investors in protecting their investments from a potential loss to occur. In this study, the stock markets' behaviour was demonstrated, as well as, its route that was started from the state of which, there was no crisis until reaching the state of which the crisis has reached its peak. The analysis has shown that, for the first time, the distinction between the markets, which are potentially falling into a crisis and the ones which are not, can be made. In addition, the transition from one state (no crisis) into another state (crisis time) was observed and detected.

A general conclusion of the results discussed Sections 3.1 and 3.2, is presented in Figure 3. Figure 3 shows that there are two regions: (i) a green region represents the state when there was no crisis and, (ii) a red region represents the state when the crisis has reached its peak. These results allow for the first time in the field of stock market analysis, investors and market players the warning sign which provides them the possibility of tracking which markets are going through a potential crisis. This differentiation allows them to avoid investing in such markets and going away from a potential catastrophic to occur. In other words, the breakthrough results shown in this manuscript are expected to be of aid for investors in improving the process of decision making in portfolio selections. This allows them to reduce risk exposure associated with their portfolios. More specifically, according to these results, investors can then exclude or withdraw their investments from the companies that are expected to go through a potential crisis. This is done to protect their investments from a certain loss.

Such analysis can not only be done for financial markets, but also for other systems. For example, in Neuroscience recognizing certain patterns promises early warnings, in order to discover Brain diseases is one of the main objectives in such field. Another example can be seen is when studying climatic changes to observe and detect certain patterns that can be useful in predicting a potential catastrophic.

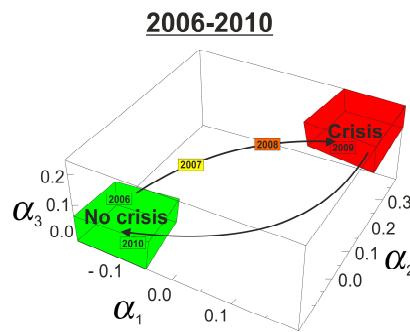


Figure 3: An Illustrative Figure that Shows the Two States, Which are No-Crisis and Crisis State, Where the Markets will be Most Likely are. The Directed Lines Reflect the Direction of the Trajectories of the Stock Markets

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